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Tsan-Ming Choi *Editor*

# Handbook of EOQ Inventory Problems

## Stochastic and Deterministic Models and Applications

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# Preface

Inventory management is a critical factor which accounts for the success or failure of modern businesses in all kinds of industries. By far the best known inventory model is the classical "square root formula" of the economic order quantity (EOQ) model. The widely recognized first piece of research on the EOQ model appeared a century ago in Harris (1913) which describes "a very simple deterministic inventory planning model with a tradeoff between fixed ordering cost and inventory carrying cost." Despite being simple, this model does capture the essence of inventory management and lays the foundation for all kinds of extensions and real-world applications (see Axsäter 1996; Huang et al. 2003; Khan et al. 2011; Pentico and Drake 2011). Nowadays, a search in the major research portals will find at least a thousand papers which carry the key words of "EOQ" in the paper title. In fact, tens to hundreds of related papers are still being published in major journals in operations research and management science every year. Despite the abundance of both classical and new research results, there is an absence of a comprehensive reference source that provides the state-of-the-art findings on both theoretical and applied research on EOQ and its related models. As a result, I organize this Springer's handbook with a goal of consolidating many latest research findings and applications of the EOQ model into an edited volume. I believe that this handbook will be a pioneering text focusing on the EOQ model-related inventory and supply chain management problems. It also celebrates the EOQ model's 100th anniversary.

The handbook contains papers which explore both the deterministic and the stochastic EOQ model-based problems and applications. It is organized into three parts: Part I presents the introduction and review papers. Part II includes technical analyses on single-echelon EOQ model-based inventory problems. Part III consists of applications of the EOQ model for multi-echelon supply chain inventory analysis. I am very pleased to see that this handbook has generated a lot of important insights and new research results on the EOQ model-related problems.

I would like to take this opportunity to show my hearty gratitude to Fred Hillier and Matthew Amboy for their kind support and advice along the course of carrying out this book project. I sincerely thank all the authors who have contributed their interesting research to this handbook. I am indebted to the anonymous reviewers who reviewed the manuscripts and provided me with very constructive and timely review comments. I also acknowledge the editorial assistance of my Ph.D. student Ms. Hau-Ling Chan, and the funding support of the Research Grants Council of Hong Kong under grant number PolyU 5424/11H (General Research Fund). Last but not least, I am grateful to my family, colleagues, and students, who have been supporting me during the development of this important research handbook.

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# Part I
# Introduction and Review

# A Century of the EOQ

**Matthew J. Drake and Kathryn A. Marley**

**Abstract** When Ford W. Harris published his short three-page article developing the Economic Order Quantity (EOQ) model in 1913, he likely did not foresee that it would still be discussed and used 100 years later. Harris' EOQ model was one of the first applications of mathematical modeling to guide managers in making business decisions, and it has spawned thousands of related studies over the past century that have built on its major foundations and insights. In this chapter we present a short history of the EOQ model by discussing the model itself, some practical issues about implementing the model, and major extensions to the basic model grouped by the dominant foci of each subsequent decade.

## 1 Introduction

Every organization must determine the number of items or units to order every time it acquires stock from its suppliers. Perhaps it is this universal application to every type of business that has kept the Economic Order Quantity (EOQ) model relevant for 100 years. First published by Ford W. Harris in 1913, the EOQ model prescribes the optimal order quantity for organizations that minimizes the total ordering and holding cost under a relatively restrictive set of assumptions.

Even with these restrictions, it is impossible to overstate the influence that the EOQ model has had on a century of researchers and practitioners in the fields of operations management and operations research. This is largely because the model is the foundation for literally *thousands* of later studies that relaxed a subset of its

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assumptions to create a modified model that better fits a particular decision environment. The remaining chapters in this book highlight cutting-edge research in the field by including today's contemporary extensions of the original model. The EOQ model is also an essential part of the history of operations research because it represents one of the first published applications of a mathematical model to business decision making.

In addition, the original EOQ model in its unadjusted form remains relevant because it is widely used in practice. Undoubtedly, some organizations apply the model incorrectly to situations where it is not the best practical solution, but it still works well in practice because of its simplicity and robustness (Ptak 1988), which are discussed in detail in the next section of the chapter. The model is also taught in a majority of academic courses that cover inventory control to even a small degree. It proves to be an effective method to expose students and practitioners to the major cost trade-off in inventory control—ordering costs versus holding costs.

The remainder of the chapter is organized as follows. The next section discusses the original EOQ model itself along with its assumptions and the practical issues with implementing the model in a business setting. The subsequent five sections detail the development of EOQ-based research decade by decade from the 1950s to the present. Concluding remarks are provided in the final section of the chapter.

## 2 The Original EOQ Model

### *2.1 Model Assumptions*

Harris' original EOQ model in 1913 was established to provide a guideline for managers to use when ordering items from their suppliers. Technically, his article says that it determines manufacturing quantities, which is evident in the refreshingly straightforward title of the paper ("How Many Parts to Make at Once"); however, his model really only applies to *batch* manufacturing where all units become available to satisfy customer demand all at once in contrast to items produced one at a time on an assembly line. The Economic Production Quantity (EPQ) model, which determines the optimal batch size for this one-at-a-time type of production where the first units in a batch can be used to fulfill customer orders while the rest of the batch is still being produced, is one of the first examples of extensions to the original EOQ model (Taft 1918). The assumption of simultaneous availability of the entire order quantity is appropriate for situations where organizations purchase items from suppliers because they usually arrive in complete transportation batches, but the EPQ model is more applicable to single-unit, assembly line production environments.

In addition to the simultaneous availability assumption, the original EOQ model assumes the following conditions. The notation used in the model is also defined in the list below.

- Annual demand for the item, $D$, is deterministic and occurs at a constant rate over time. This assumption is especially problematic in situations where demand varies from month to month or from season to season throughout the year. Silver et al. (1998) argue that if the variation of demand from period to period is sufficiently low (i.e., the squared coefficient of variation is less than 0.2), the original EOQ model with constant demand can be used without a large degradation in results; when the variation in demand from period to period is high, firms should utilize a model that considers this variation such as the Wagner-Whitin algorithm discussed in Sect. 3.
- The unit cost of the item, $p$, is known and fixed over the length of the planning horizon. This is a reasonable assumption in practice when firms have negotiated a long-term, fixed-price contract for the item. Extensions of the original EOQ model exist for quantity discount situations or items subject to significant inflation. Many of these models are discussed in the subsequent sections of the chapter.
- Lead times for receiving orders, $L$, are known and constant. This assumption applies in practice when the firm has a high-quality supplier that fulfills orders consistently within the same period of time. This situation becomes less applicable in practice when suppliers are located far away from the company because the shipments spend more time in transit and are subject to more uncertainty with respect to transportation conditions and customs scrutiny at international borders.
- The firm's ordering cost, $S$, is fixed and independent of the size of the order quantity.
- The firm's annual holding cost rate, $r$, is fixed and independent of the size of the order quantity. Thus, the cost of holding a unit in inventory for an entire year, $H$, can be computed as $H = r^*p$.
- No capacity or financial limitations apply for the firm or its supplier. This is especially applicable for make-to-stock products that are available immediately in a supplier's distribution center as well as for cheap items for which the firm has ample cash reserves to pay for orders.
- No stockouts are allowed; that is, the firm orders enough items to satisfy all of the demand when it occurs. It can be shown, however, that if the cost of backordering is sufficiently low, the firm can reduce its total cost by planning to backorder some of the demand (Zipkin 2000).

## 2.2 Model Derivation

In light of these assumptions, the firm seeks to determine the optimal order quantity, $Q^*$, which minimizes its total annual relevant costs. No quantity