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Optimal Analysis of Structures by Concepts of Symmetry and Regularity

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Foreword

Recent advances in structural technology require greater accuracy, efficiency and speed in the analysis of structural systems, referred to as *Optimal Analysis of Structures Using Concepts of Symmetry and Regularity*. It is, therefore, not surprising that new methods have been developed for the analysis of structures with a large number of nodes and members.

The requirement of accuracy in analysis has been brought about by the need for demonstrating structural safety. Consequently, accurate methods of analysis had to be developed, since conventional methods, although perfectly satisfactory when used on simple structures, have been found inadequate because of the requirement of high computational effort for large-scale structures. Another reason why greater accuracy is required is the need to achieve efficient and optimal use of the material, i.e. optimal design.

In this book, different mathematical concepts are combined to make the optimal analysis of structures feasible. Canonical forms from matrix algebra, product graphs from graph theory and symmetry groups from group theory are some of the concepts involved in the variety of efficient methods and algorithms presented.

The methods and algorithms elucidated in this book enable the analysts to handle large-scale structural systems by lowering their computational cost fulfilling the requirement for faster analysis and design of future complex systems. The value of the presented methods becomes all the more evident in cases where the analysis needs to be repeated hundreds or even thousands of times, as is the case with the optimal design of structures using different meta-heuristic algorithms.

This book is of interest to all those engaged in computer-aided analysis and design, and also to software developers in this field. Though the methods are illustrated mainly through skeletal structures, however, some continuum models have also been added to show the generality of the methods. The concepts presented in this book are not only applicable to different types of structures, but can equally be used for the analysis of other systems, such as hydraulic and electrical networks.

The author has been involved in various developments and applications of graph theory in the last four decades. The present book contains part of this research, suitable for various aspects of matrix structural analysis.

The present book is intended to serve as a textbook for the optimal analysis of large-scale structures utilising concepts of symmetry and regularity. Special attention is focused on efficient methods for eigensolution of matrices involved in static, dynamic and stability analyses of symmetric and regular structures, or those general structures containing such components. Powerful tools are also developed for configuration processing, which is an important issue in the analysis and design of space structures and finite element models.

The book is written in an attractive dynamic style that immediately goes to the heart of each subtopic. The many worked out examples will help the reader to appreciate the theory. The book is likely to be of interest to pure and applied mathematicians who use and teach graph theory as well as to students and researchers in structural engineering and architecture.

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Professor Emeritus
Dr. Dr. h.c. Franz Ziegler

Preface

Concepts from different fields of mathematics are combined to obtain powerful tools and algorithms for efficient analysis of structures. Many structures are either symmetric or regular, and some others can be made symmetric or regular by addition or elimination of a small number of nodes and/or members. For these structures, the matrices have canonical forms and the corresponding equations can easily be solved using some concepts from matrix algebra, linear algebra, graph theory, and group theory.

The methods and algorithms developed in this book make the analysis of large-scale structures possible by reducing their computational time and storage, fulfilling the requirements for a faster analysis of complex systems. The power of the presented methods becomes more evident when analysis needs to be repeated many times, as is the case with optimum design of structures utilizing different meta-heuristic algorithms.

The author has been involved in various developments and applications of graph theory in the last four decades. The present book contains part of this research, suitable for matrix analysis of symmetric and regular structures.

Methods of this book can efficiently be used in computer-aided analysis and design, and commercial software developments. Though these methods are mainly illustrated through skeletal structures, some continuum models have also been included to show the generality of the algorithms.

The present book is intended to serve as a textbook for the optimal analysis of large-scale structures utilising concepts of symmetry and regularity. Special attention is focused on efficient methods for eigensolution of matrices involved in static, dynamic and stability analyses of symmetric and regular structures, or those general structures containing such components. Powerful tools are also developed for configuration processing, which is an important issue in the analysis and design of space structures and finite element models.

In Chap. 1, an introduction is provided to the definitions and basic concepts of symmetry and regularity. Chapter 2 presents a background of the mathematics extensively used in this book, consisting of definitions from graph theory and algebraic graph theory. Standard definitions of graph products and their extensions

are provided in Chap. 3 and utilised in the important topic of configuration processing of structures. Basic definitions of canonical forms and their properties involved in symmetric and regular structures are discussed in Chap. 4. The canonical forms are applied to two important combinatorial optimisation problems consisting of nodal ordering to improve the patterns of the structural matrices, and graph partitioning for use in parallel computing in Chap. 5. Chapter 6 utilises graph products for similar purposes as in the previous chapter. Canonical forms have important applications in structural mechanics. These applications are discussed in Chap. 7. Graph products make the efficient analysis of regular structures feasible, providing closed-form solutions for this purpose as discussed in Chap. 8. Some structural models are not regular but can be made regular by adding and/or deleting of some members. Chapter 9 contains efficient methods for eigensolution and analysis of such structures using direct methods. Iterative methods for similar purposes are presented in Chap. 10. Group theory is known as the language of symmetry. Basic concepts and applications of group theory are discussed in Chap. 11. Finally, the interrelation of canonical forms, graph products and group theory and their applications to the analysis of symmetric-regular structures are presented in Chap. 12.

I would like to take this opportunity to acknowledge a deep sense of gratitude to a number of colleagues and friends who in different ways have helped in the preparation of this book. Mr. J.C. de C. Henderson, formerly of Imperial College of Science and Technology, first introduced me to the subject with most stimulating discussions on various aspects of topology and combinatorial mathematics. Professor F. Ziegler encouraged and supported me to write this book. My special thanks are due to Mrs. Silvia Schilgerius, the senior editor of the applied sciences of Springer, for her constructive comments, editing and unfailing kindness in the course of the preparation of this book. My sincere appreciation is extended to our Springer colleagues Mr. C. Bachem and Ms. G. Umamaheswari.

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Every effort has been made to render the book error free. However, the author would appreciate any remaining errors being brought to his attention through the following email address: alikhavah@iust.ac.ir.

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