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2nd Order Singularity-free Solutions

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To
Liping, Yuxi and Pan

Preface to the Second Edition

As mentioned in the preface of the first edition of this book, the symbolic mathematical operation software design is one of the key components for deriving formulas and developing theory. During the time of this revision, Mr. Jia Xu has taken over most of the work concerning program optimization and design, results computation and validation. Due to his elegant software design, the solution of solar radiation disturbance on the satellite orbit, which was not successfully derived in the first edition, has been obtained. Through optimization and homogeneous truncation, the solution series of geopotential disturbance is now derived up to 8×8 degrees and orders which are sufficient for application of MEO and GEO satellite orbit determination. Because of the extreme importance of Jia Xu's contribution, Springer has agreed to include him as co-author of this new edition, and this is certainly well deserved.

After the first edition of this book was published at the end of 2008, the first author was very happy to put aside the hard work of writing and concentrate – with a small team – on the practical application of developed analytical satellite orbit theory. Experiences from the practice and implementation of the theory into software gave the author a strong feeling that the theory must be further developed. At the beginning of 2010, the first author started to revise and refine the theory and wrote a series of papers for books and journals for review and publication (Xu 2010, Xu et al. 2010a, b, 2011, Xu and Xu 2012, 2013c). Due to the difference in their scientific working areas, the first author very much regrets that the contribution of the second author on software design and strategic discussion is not properly acknowledged. Through the reviews of and communications with some of the top experts in celestial mechanics, the purpose of this new edition is becoming clearer: to revise and derive complete singularity-free solutions for satellite orbits disturbed by all perturbation forces (Chaps. 5, 6, 7, and 10) and to make the theory applicable (Chaps. 8 and 9). The highlight of this edition is that the singularity problem in orbital mechanics is solved for the first time cf. Chap. 10 or Xu and Xu 2012). Orbital problems in the solar gravity field are also discussed (Chap. 11). We further concentrate on the theory of second-order solutions instead of attempting to derive those of the third order. The papers published in the top journal MNRAS are

invaluable for revising and supplementing the first edition, for modifying parts of the key contents and reporting on new theoretical progress and knowledge and for laying down a solid theoretical basis for future applications. With the coming of the satellite era, analytical satellite orbit theory will surely be important for applications such as orbit determination without global coverage of the monitoring station network, long-term orbit forecasting, on-board autonomous orbit determination and state controlling and monitoring, solar radiation effects monitoring as well as space mission orbit design. The authors are grateful to the readers of the first edition, whose interest led Springer to request for this second edition as early as 2010.

Compared to the book *GPS – Theory, Algorithms and Applications* which took more than 2 years to write the first edition (Xu 2003) and 9 months to revise for the second edition (Xu 2007), only 6 months of extreme hard work were needed for the first edition of this book *Orbits* (Xu 2008); however, more than 2 years have already been expended in preparing this second edition. The authors hope that this very extensively revised and supplemented edition will be complete in theory yet easy to use in applications. Worth mentioning is that there is a review of the first edition of this book which is published in MathSciNet (MR2494776) of the American Mathematical Society written by Pierre Rochus, professor and associate director of Spatial Centre of the University of Liege. He stated to the author through an email on 11 August 2009: "In my modest opinion, I think that your book is very original. I read a lot of books on Celestial Mechanics (I gave lectures on Celestial Mechanics at the University of Liege for 15 years) and your book brings new ideas". In this revised version, complete solutions on all possible disturbances are given in detail with applicable and comprehensive methods.

The analytical solution of the C_{20} disturbance discussed in Chap. 5 is greatly revised by taking into account the geopotential coefficient with a special truncation order of eccentricity. The analytical solutions of the geopotential disturbance up to 8×8 degrees and orders are derived and given in Chap. 6 and Appendix 2. In Chap. 7, one of the key contents of this book concerning extraterrestrial disturbances is most intensively revised by taking into account the theoretical parts of the papers by Xu et al. (2010a, b) and Xu and Xu (2012), by using so-called basic Lagrangian and Gaussian equations of motion and different truncations, so that simpler and more precise formulas are obtained. The solutions of all kinds of disturbances are derived in a similar but independent way. Solutions of extraterrestrial disturbances such as solar radiation pressure, atmospheric drag and disturbances by the sun, moon and planets are given. Application of orbital analytical solutions is discussed in more detail. An additional chapter (Chap. 11) is presented for orbit mechanics in the solar gravity field, including solar oblateness disturbance (Xu et al. 2011) for planetary perihelion precession and solar radiation disturbance (Xu and Xu 2013c) for modification of Kepler's third law through comparison with traditional mass correction results. A singularity-free theory is also developed (Chap. 10) based on the research described in the first edition of this book and a review of theoretical orbit theory (Xu 2010) as well as an intensive study of Xu and Xu (2012). A summary is given for the so-called Lagrange-Xu and

Gauss-Xu equations of motion by using basic Lagrangian and Gaussian equations. How to form the singularity-free solutions based on solutions of the basic equations and how to solve the critical inclination problem are described in detail with examples and new defined criteria (Chap. 10).

The extended contents have been reviewed by individual and anonymous journal reviewers. We thank the reviewers for their thought-provoking comments and valuable suggestions: Prof. Rudolf Dvorak of Vienna University, Prof. Ludwig Comblinck of Hertro Astronomical Observatory, Prof. Rolf Romer and Dr. Svetozar Petrovic of GFZ and Prof. Zhiping Lü of the Institute of Surveying and Mapping in Zhengzhou. We also thank Prof. Pierre Rochus of Liege University in Belgium for reviewing the first edition of this book (Rochus 2009).

The first author wishes to thank sincerely Prof. Wu Chen of the Polytech University of Hong Kong, Prof. Ta-Kang Yeh of the Taipeh University of Taiwan, Prof. Yuanxi Yang of the Institute of Surveying and Mapping (ISM) in Xi'an, Prof. Qin Zhang of ChangAn University (CAU), and his team members, scientist Dr. Tianhe Xu of GFZ and ISM, PhD candidate Xin Shen of GFZ and Wuhan University, Keifei He of GFZ and Technical University of Berlin, Guanwen Huang of GFZ and CAU, Xiao Cao of GFZ and Chinese Academy of Space Technology and Yan Xu of CAU and GFZ for their cooperation and discussion as well as co-authorship in several scientific journal papers. Theoretical progress of the paper by Xu et al. (2010a) has been stimulated by an application report of Dr. Tianhe Xu, which was one of the key contributions during his research time in GFZ. The second author, Jia Xu, is especially thanked for constant support and cooperation over the years. We co-authored the significant paper "On the singularity problem in orbital mechanics" Xu and Xu (2012) which solves for the first time the singularity problem in orbital mechanics. We are now inseparable cooperators on a completely new and original book *N-Body – Solar System Analytical Solution* from Springer (contracted), in which we will present for the first time the analytical solutions of the N-body problem in our solar system, and a further software companion book *GPS – Software Companion* from Springer (contracted) for the third edition of *GPS – Theory, Algorithms and Applications* (contracted).

The first author wishes to thank sincerely his former directors Prof. Dr. Christoph Reigber and Prof. Dr. Markus Rothacher and new director Prof. Dr. Harald Schuh for their support and trust during their time at the GFZ and for granting him great freedom of research, without which some of the scientific activities would never have been possible to realise. He is also grateful to the acting heads of the sections of GFZ, Dr. Christoph Förste, Dr. Frank Flechtner and Dr. Jens Wickert, for their support. He also thanks the Chinese government for honouring him with the title of National Distinguished Expert, owing to which he has been fortunate enough to have had the involvement of the Chinese Academy of Space Technology for research in GNSS navigation and orbital mechanics as well as in space missions. He thanks the Southwest Jiaotong University for honouring him with the Honorary Professor title. He is also grateful to his many close friends for their friendship and support. His special thanks go to Prof. Jörg Reinking of Jade University of Applied Sciences, Dr. Ludger Timmen of University of Hannover,

Prof. Ludwig Comblinck of Hertro Astronomical Observatory, Prof. Pierre Rochus of Liege University, Prof. Yuanxi Yang of ISM in Xi'an, Prof. Qin Zhang of CAU, Prof. Zhilin Li of the Polytech University of Hong Kong, Prof. Zhiping Lü and Prof. Shulong Zhu of the Institute of Surveying and Mapping in Zhengzhou, Dr. Gim Der of DerAstrodynamics, Prof. Yunzhong Shen of Tongji University and Dr. Hong He of Helmholtz's representative office in Beijing. He is also grateful for the valuable feedback from readers and from students through his professorships at ChangAn University and the NTSC CAS as well as Neubrandenburg University of Applied Sciences.

May 2012 Guochang Xu and Jia Xu

Preface to the First Edition

The purpose of this reference book is to describe and to derive the analytical solutions of the equations of satellite motion perturbed by extraterrestrial and geopotential disturbances of the second order. The equations of satellite motion perturbed by extraterrestrial disturbances are solved by means of discretization and approximated potential function as well as Gaussian equations. The equations perturbed by geopotential disturbances are solved by symbolic mathematical operations. The traditional problem of singularity in the solutions is solved by so-called singularity-free orbit theory. Simplified disturbed equations of motion are proposed to simplify the solutions. Applications of the theory for analytical orbit determination are also discussed. Indeed, this is the first book since the satellite era, which describes systematically the orbit theory with analytical solutions, with respect to all of extraterrestrial and geopotential disturbances of the second order, and the solutions are free of singularity. Based on such a theory, the algorithms of orbit determination can be renewed; deeper insight into the physics of disturbances becomes possible; the way to a variety of new applications and refinements is opened.

My primary knowledge of the orbit theory came from my education of mathematics while studying physics and theoretical mechanics (1981). My first practical experience with orbits came from the research activity at the Technical University (TU) Berlin on orbit corrections of satellite altimetry data (1988–1992). The extensive experience on orbit came from the GPS/Galileo software development for orbit determination and geopotential mapping at the GFZ (2001–2004). The traditional adjustment model of the solar radiation used in numerical orbit determination is investigated and considered not reasonable physically; and a new adjustment model is proposed in the user manual of the Multi-Functional GPS/Galileo software (MFGsoft) (Xu 2004), which is also reported in the 2nd edition of the book "*GPS – Theory, Algorithms and Applications*" (Xu 2007). Indeed, one of the ways to obtain the solutions of the extraterrestrial disturbances of the satellite motion is found during that investigation. However, it has not been realised until two scientists, Dr. Xiaochun Lu and Dr. Xiaohui Li of the National Time Service Center (NTSC) in Xi'an, came to visit and to cooperate with me at GFZ. We discussed the

virtual navigation system and tried to solve the stability problem of the 3-D positioning of the system. By considering what is significant in theory and, what is more important than our numerical study, the idea of solving the disturbed equations of motion was obtained, and the solutions of the extraterrestrial disturbances of the equation of satellite motion were found. Because of the importance of the geopotential disturbances, great efforts were then made to derive the related solutions. Thereafter, alternative solutions of the extraterrestrial disturbances were found by using different means (besides the discretization, also approximated potential function and Gaussian disturbed equations). To simplify the solutions, the simplified disturbed equations were proposed. To solve the problem of singularity, the singularity-free theory was also developed.

After publishing my book, "*GPS – Theory, Algorithms and Applications*", in 2003, I did not want to ever write another scientific book because this process took more than 2 years of extremely hard work. However, I must finish this book because some of the scientists have contributed their lifetime to the theoretical solutions of the geopotential disturbances of the equation of satellite motion and now the results are here. The solutions of the extraterrestrial disturbances of the orbit motion are of extreme importance for practice, but they are rarely investigated because they are highly complex. From the theory, a special confusion related to the solar radiation from the pure numerical orbit determination has been cleared. Many interesting applications will follow soon. To make the process of writing easy, a small portion of the basic contents of my GPS book is partly modified and imported or rearranged and used.

The book includes ten chapters. After a brief introduction, the coordinate and time systems are described in the second chapter. The third chapter is dedicated to the Keplerian satellite orbits – the orbits of the satellite under the attraction of the central force of the Earth.

The fourth chapter deals with perturbations of the orbit. Perturbed equations of satellite motion are derived. Perturbation forces of the satellite motion are discussed in detail, including the perturbations of the Earth's gravitational field, Earth tide and ocean tide, the sun, the moon and planets, solar radiation pressure, and atmospheric drag, as well as coordinate perturbation.

The fifth chapter covers the analytical solution of $\bar{C}_{20}$ perturbation, including the complete formulas of the long term, and long and short periodic terms. The derivation also gives the algorithm and model of orbit correction. The solutions of other geopotential disturbances of higher order and degree are described in the sixth chapter. As examples, solutions of disturbances of $\bar{C}_{30}$, D_{21} and D_{22} are given. General solutions of disturbance of D_{lm} are derived. Symbolic operation software for deriving solutions of geopotential disturbances of any order and degrees are designed and used.

The seventh chapter covers the solutions of extraterrestrial disturbances such as solar radiation pressure, atmospheric drag and the disturbances of the sun, the moon and planets. The principle and strategy that lead to the solution are described. The solutions are derived via discretization and approximated potential function as well

as Gaussian perturbed equations of motion. Simplified disturbed equations are proposed and used partly. The ephemeris of the sun, the moon and planets are given for practical use.

The eighth chapter is dedicated to numerical orbit determination, including its principle, the algebraic solutions of the variation equations, and the numerical integration and interpolation algorithms, as well as the related derivatives.

The ninth chapter describes the principle of analytical orbit determination based on the proposed new solutions. Real time ability and properties of the analytical orbit solutions are discussed.

The final chapter includes algorithms that lead to singularity-free orbit theory and the equations of motion in non-inertial frame as well as discussions concerning the further development of the orbit theory and its applications as well as comments on some remaining problems.

The book has been subjected to an individual review of chapters and sections and a general review. I am grateful to reviewers Prof. Markus Rothacher of GFZ, Prof. Dieter Lelgemann of TU Berlin, Prof. Yuanxi Yang of the Institute of Surveying and Mapping (ISM) in Xi'an, Dr. Jianfeng Guo of Information Engineering University (IEU) in Zhengzhou, Prof. Xuhai Yang of NTSC in Xi'an, Dr. Junping Chen of GFZ. A grammatical check of technical English writing has been performed by Springer Heidelberg.

I wish to thank sincerely Prof. Markus Rothacher for his support and trust during my research activities at GFZ. Dr. Jürgen Kusche is thanked for his encouragement and help. Prof. Dr. Christoph Reigber is thanked for granting me special freedom of research. My grateful thanks go to Dr. Xiaochun Lu and Dr. Xiaohui Li of NTSC in Xi'an. Their visit to and cooperation at the GFZ have led to the derivations of the key contents of this book. Dr. Jiangfeng Guo of IEU in Zhengzhou followed a part of my derivation and checked it for correctness. Volker Grund of GFZ helped me greatly by assisting in the application of software tools, which is another key to the solution of geopotential disturbances. Qianxin Wang of GFZ helped to check some of the formula typing. Dr. Jinghui Liu of the educational department of the Chinese Embassy in Berlin, Prof. Yuanxi Yang of ISM in Xi'an, Prof. Heping Sun of the Institute of Geodesy and Geophysics (IGG) in Wuhan and Prof. Qin Zhang of ChangAn University in Xi'an are thanked for their friendly support during my scientific activities in China. The Chinese Academy of Sciences is thanked for the Outstanding Overseas Chinese Scholars Fund, which supported greatly many valuable scientific activities even outside China.

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