

Intelligent Systems Reference Library 67

Oleg Granichin
Zeev (Vladimir) Volkovich
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Randomized Algorithms in Automatic Control and Data Mining

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Series editors

Janusz Kacprzyk, Polish Academy of Sciences, Warsaw, Poland
e-mail: kacprzyk@ibspan.waw.pl

Lakhmi C. Jain, University of Canberra, Canberra, Australia
e-mail: Lakhmi.Jain@unisa.edu.au

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Oleg Granichin · Zeev (Vladimir) Volkovich
Dvora Toledano-Kitai

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Oleg Granichin
Department of Mathematics and Mechanics
Saint Petersburg State University
St. Petersburg
Russia

Dvora Toledano-Kitai
Department of Software Engineering
ORT Braude College
Karmiel
Israel

Zeev (Vladimir) Volkovich
Department of Software Engineering
ORT Braude College
Karmiel
Israel

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Preface

The authors start their book with basic question: *Why is randomization beneficial in the context of algorithms?* Or, say it another way: *When random choice is better than deterministic one?* This more general problem relates to a lot of situations beyond the mathematical framework. One can remember that random decisions were performed in ancient times, and the procedure of drawing lots was very common. Moreover political events such as election of governing officers in Athens were randomized. In our pragmatic age the field of random decisions in everyday life became narrower, but surprisingly mathematical algorithms opened the door for randomness. One of the first examples is the mixed strategy in two-person games, invented by von Neumann. However in the situations under consideration there is no “enemy” to mislead, the randomness is introduced to accelerate algorithms or make them more reliable. The first technique of this sort is the famous Monte-Carlo method.

The authors carefully examine numerous applications of randomized algorithms in statistics, optimization, control, data mining, machine learning. They demonstrate serious advantages of the algorithms in various cases and explain the reasons. Many of the proposed methods are new and are of wide interest. If compared with some other books on randomized algorithms, this monograph is not so specialized and is devoted to very broad field of problems. One of the peculiarities of the monograph - it summarizes deep researches in Russian-language literature which are not widely known to the Western audience.

I believe that the readers will find the book “Randomized Algorithms in Automatic Control and Data Mining” both fascinating and useful.

Boris Polyak,
Institute for Control Science, Moscow
IFAC Fellow
February 2014

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Introduction

In 1948 Norbert Wiener's famous book proclaimed the establishment of a new science of CYBERNETICS in which the information-control relation in the phenomena of the material world plays the role of its fundamental property.
Vladimir Fomin, 1984 [118]

This text contains an exposition of randomized algorithms and their applications, thus providing some answers to the question: *Why is randomization beneficial in the context of algorithms?*

From Scalability toward Adaptability: Perspectives of Computing

In the new field of computers, the Olympic motto “Citius, Altius, Fortius” has been transformed into a new slogan: “Fast, Powerful, Miniature”. In fact, these ostensibly conflicting goals join together into one: the technology approaching the creation of “mobile” artificial intelligence. Soon intelligent embedded computing devices will be able to perform such functions that computer users never even “dreamed” of a few years ago.

Throughout the history of civilization, there have been several information revolutions — transformation of social relations based on the dramatic changes in information processing and information technology. In each case, new qualities of the human community were achieved every time as consequences of these changes.

The first information revolution began with the separation of human from nature, as evidenced in rock painting and the appearance of the language that was able to handle abstract concepts. The invention of writing knowledge to be extended and preserved for transmission to future generations.

The next stage was the invention of printing, which radically changed the society and culture. Books became the mass universal distributor and the custodian of large volumes of information.

The era of electricity brought with it the capabilities of telegraph, telephone, radio and television that allow information to be sent quickly to any corner of the Earth.

In the wake of the Second World War and post-war economic development, research studies in nuclear and micro-molecular physics, solid state electronics and border effects led to the creation of the first industrial computing devices, supporting the growth of a new industrial revolution. For a quarter century these developments paved the way for a rapid burst of information technologies.

When the universal, multi-functional, automatic electronic devices known as computers were invented, they took over the majority of data processing, filing and recording functions.

Let's look at the history of the development of computer technology. For six decades it has paved the way from electronic lamps through transistors and integrated circuits to very large-scale integrated circuits. What will happen next? The main question is how to handle the huge amount of data being generated?

At the beginning of the twenty-first century we see that the technology-driven complexity of economy and society is increasing. During the last quarter of the twentieth century the human community was still an urban society with an industrial economy and mass products technology, but now it is increasingly characterized as a global society with a knowledge economy, and digital technology. Earlier, *a key resource* was capital, but now it is knowledge/information that dominates. Where earlier *distribution* focused early on a motorways, now the digital networks have come to the forefront. Previously the *scope* was a regional but now it is mainly global. Economy of scale, the undisputable key *success factor* the industrial economy, is less and less important as the complexity (and dynamics) of the knowledge economy increases. The new key success factor is ADAPTABILITY, which is the ability to rapidly produce a positive response to unpredictable changes [280].

People have always been interested in artificial intelligence. By the late twentieth century, the modeling of artificial intelligence was marked by two main approaches:

- machine intelligence, involving the setting of strict results of operations;
- artificial mind, based on the modeling of the internal structure of automated systems to correspond to the structure of a human brain.

But as of today, humanity has made a major move from the theories to a new reality: the cybernetic future. Many signs indicate that we have now entered a new phase — *a cybernetic revolution*.

New requirements and the challenges of globalization along with the exponential increase in the complexity of computing systems have already pushed us to think in practical terms about the prospects and possible changes in the computing paradigm:

“What is the process of computing?”

Today’s objective trends are toward miniaturization and toward improvement of processor performance. These trends have brought technology to the threshold of traditional computing devices, as was predicted by Moore’s Law [239]. Manufacturers are moving from priorities of increasing clock frequency and the power of one CPU toward multi-core processors, parallelism and so on.

Indeed, the standard for laptops today is to use multi-core processors, and, of course, supercomputer processors have many more cores. Now that “the genie is out of the bottle”, there must be consequences. Soon systems will have dozens, and then thousands, of cores. Completely new architectures will emerge. Cores will be combined into complex blocks, different computing clusters will have parallel and simultaneous access to data, and computing units will communicate through a common memory. In fact, many aspects of the computing paradigm will change, including the nature of computing devices and of computational processes.

The traditional understanding of what is inside a computer and what constitutes a computing system will also change. These changes will lead to transformations in programming style and in the way in which computational devices are used.

The transition to a new paradigm of computing will probably cause the architecture of computing devices to shift toward a set of concurrent asynchronous models of interacting dynamical systems (functional elements). The properties of stochastic, hybrid, asynchronous and cluster behavior (among them the absence of rigid centralization and the dynamic clustering into classes of related models) will be more apparent and dominant among the new features of the future paradigm.

Stochasticity. It is well known that computers are becoming smaller and smaller. The size of an elementary computational element (a gate) now approaches the size of a molecule or an atom. At this scale, the laws of classical physics are not applicable and quantum laws begin to act, which, due to Heisenberg’s uncertainty principle, conceptually do not give precise answers about a system’s current state. On the other hand, stochasticity is a well known property of complex dynamical systems comprising a large number of components.

The *hybrid* nature of future computing necessitates the examination of a combination of continuous and discrete processes, i.e., registering the continuous evolution of physical processes during the work of this or that model

and abruptly switching from one model to another. The increase in the speed of computing devices and the reduction in their size inevitably lead to the need for operations with “transitional” processes. A serious limitation of the classical computation model is the separation of memory into isolated bits. From a certain level, the reduction in the length of a clock cycle time (strobe impulse) and in the distance between the bits makes it impossible to consider bits to be isolated, due to the operation of the laws of quantum mechanics. In the future it will be natural to switch from primitive operations with classical bits to operations definable by certain micro-dynamic models that operate with sets of related “bits”. In this case, classical operations with bits may continue to be as simplest “models”.

Success in solving traditional complex multidimensional problems (such as new algorithms working “per clock cycle”) is the rationale for examining a wider class of models. Often it is possible to get an answer as a result of a physical adiabatic process. For example, the classical operation on bits is the transition of a physical system (trigger) from state “1” to “0”. P. Shor suggested the quantum Fourier transform algorithm which can be performed for a time proportional to $(\log_2 N)^2$ and not for $N \log_2 N$, like the classical fast Fourier transform [293]. For the 10th Hilbert problem, [316] discusses solving the hypothetically possible “physical” method. The considered approach is based on the quantum adiabatic theorem and the algorithm works finite time. In [325] the powerful quantum algorithm was proposed for “per clock cycle” computation of efficient estimation of the gradient vector of the multidimensional function defined with a high degree of uncertainty. The operations typical of mathematical algorithms such as functions convolution can fully be found “in nature”. Recent studies of similar models show that, due to the inherent nature of the capacity for self-organization, their performance is not necessarily separated into simpler computing blocks, i.e. they cannot always be written in the form of classical algorithms. One of the possible examples of an “analog” implementation of a function convolution using a large regular array of quantum dots with typical sizes of up to 2 nm can be found in [147].

Asynchrony. The refusal to use the standardized simple computing primitives inevitably leads to the refusal to synchronize the work of various components having significantly different physical characteristics and their own internal durations of “clocks”. Within the framework of the classical set theory a controversial interpretation of the unified “clock cycle” concept is expressed in the insolubility of the problem of the continuum in the terms of Zermelo-Fraenkel axioms.

Clustering. Among the unexpected results of numerous attempts at developing complex stochastic systems (the creation of an adequate description of behavior and control) is the promising multi-agent systems model, in which the agents’ connections topology changes in time. In this case, the notion of an agent may match both some dynamical model (a system component) and a specific set of models. In the absence of rigid centralization, such