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Yeol Je Cho
Department of Mathematics Education
and the RINS
Gyeongsang National University
College of Education
Jinju, Korea, Republic of South Korea

Themistocles M. Rassias
Department of Mathematics
National Technical University of Athens
Athens, Greece

Choonkil Park
Department of Mathematics
Hanyang University
Seoul, Korea, Republic of South Korea

Reza Saadati
Department of Mathematics
Iran University of Science and Technology
Tehran, Iran

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To S.M. Ulam for the 75th anniversary of his stability problem for approximate homomorphisms

Preface

The main purpose of this book is to present some of the old and recent results on homomorphisms and derivations in Banach algebras, quasi-Banach algebras, C^*-algebras, C^*-ternary algebras, non-Archimedean Banach algebras, and multi-normed algebras.

In 1940, S. M. Ulam [321] proposed a stability problem on group homomorphisms in metric groups. In 1941, D. H. Hyers [133] proved the stability of additive mappings in Banach spaces associated with the Cauchy equation. In 1978, Th. M. Rassias [267] proved the stability of $\mathbb{R}$-linear mappings associated with the Cauchy equation, and in 2002 C. Park [220] proved the stability of $\mathbb{C}$-linear mappings in the spirit of Hyers, Ulam, and Rassias in Banach modules. Homomorphisms and derivations in Banach algebras, quasi-Banach algebras, C^*-algebras, C^*-ternary algebras, non-Archimedean Banach algebras and multi-normed algebras are additive and $\mathbb{R}$-linear or $\mathbb{C}$-linear, and so we study the stability problems for additive functional equations and additive mappings. Using the direct method and the fixed point method, the authors have studied the stability and the superstability of homomorphisms and derivations in Banach algebras, quasi-Banach algebras, C^*-algebras, C^*-ternary algebras, non-Archimedean Banach algebras, and multi-normed algebras, which are also associated with additive functional equations and additive functional inequalities.

The book provides a survey of both the latest and new results especially on the following topics:

(1) Stability theory for several new functional equations in Banach algebras and C^*-algebras via fixed point method and direct method.
(2) Stability theory for several new functional inequalities in Banach algebras and C^*-algebras via fixed point method and direct method.
(3) Stability theory of well-known new functional equations in non-Archimedean Banach algebras and non-Archimedean C^*-algebras.
(4) Stability theory for several new functional equations and functional inequalities in multi-Banach algebras and multi-C^*-algebras via fixed point method and direct method.

The book is intended to be accessible especially to graduate students who have a basic background with operator theory, functional analysis, functional equations, and analytic inequalities including an introduction to Banach algebras, quasi-Banach algebras, C^*-algebras, C^*-ternary algebras, non-Archimedean Banach algebras, and multi-normed algebras.

In Chap. 1, we provide a brief introduction to concepts with historic remarks for functional equations and their stability and the definitions of Banach algebras, quasi-Banach algebras, C^*-algebras, C^*-ternary algebras, non-Archimedean Banach algebras, and multi-normed algebras.

In Chap. 2, we study the stability of additive functional equations in Banach spaces as well as the stability and the superstability of isomorphisms, homomorphisms, derivations, and generalized derivations in Banach algebras and quasi-Banach algebras associated with additive functional equations.

In Chap. 3, we study the stability and the superstability of isomorphisms, homomorphisms, and derivations in C^*-algebras, Lie C^*-algebras, and JC^*-algebras, as well as the stability and the superstability of linear mappings in Banach modules over unital C^*-algebras. Moreover, we study Jordan $*$-derivations, quadratic Jordan $*$-derivations, (α, β, γ)-derivations on Lie C^*-algebras, square root functional equations, 3rd root functional equations, and positive-additive functional equations.

In Chap. 4, we study the stability of $\mathbb{C}$-linear mappings in Banach spaces and linear mappings in normed modules over a C^*-algebra as well as the stability of homomorphisms and derivations in proper CQ^*-algebras associated with functional inequalities.

In Chap. 5, we study the stability and the superstability of C^*-ternary homomorphisms, C^*-ternary derivations, C^*-ternary 3-homomorphisms, and C^*-ternary 3-derivations in C^*-ternary algebras as well as investigate the stability of JB^*-triple homomorphisms and JB^*-triple derivations in JB^*-triples by using the direct method and the fixed point method.

In Chap. 6, we study the stability of linear mappings in multi-Banach spaces as well as the stability and the superstability of isomorphisms, homomorphisms, and derivations in multi-Banach algebras, multi-C^*-algebras, proper multi-CQ^*-algebras, and multi-C^*-ternary algebras. Moreover, we study the stability of ternary Jordan homomorphisms and ternary Jordan derivations in multi-C^*-ternary algebras.

Finally, in Chap. 7, we study the stability of additive functional equations in non-Archimedean Banach spaces as well as the stability of homomorphisms and derivations in non-Archimedean C^*-algebras and non-Archimedean Lie C^*-algebras.

Jinju, South Korea — Yeol Je Cho
Seoul, South Korea — Choonkil Park
Athens, Greece — Themistocles M. Rassias
Tehran, Iran — Reza Saadati
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Yeol Je Cho
Choonkil Park
Themistocles M. Rassias
Reza Saadati

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